Photoabsorption in a plasma in a high magnetic field

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Photo-absorption in fully ionized plasmas in high magnetic fields is re-examined, using the methods of many-body quantum field theory. For frequencies in the immediate vicinity of the electron cyclotron resonance the rates we obtain disagree markedly from those in the literature. The new element in our work that causes most of the disagreement is the inclusion of the lowest order real part of the energy-shift of the resonant state, where, in effect, previous authors had included only the imaginary part.

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Photoabsorption in a plasma in the presence of a strong magnetic field has been addressed many times in the literature [1]-[4]. For the case of the dipole approximation to the absorption amplitude, and with the photon taken to be coupled only through the electron current, complete results can be found, for example, in ref. [1]. Recently there has been attention given to the effects of the proton current in a hydrogen plasma [5]-[11]. This becomes important in regions of extreme field and low temperature $\omega_{cp}/T \leq 1$, where ω_{cp} is the proton cyclotron frequency; that is, when thermal photons have energy in or below the proton resonance region. We will follow others' terminology and call this the region of quantized protons. The present paper has three main objectives:

a. To point out a complication that applies in either of the electron or proton resonance regions. Qualitatively we describe it as follows: In the resonance region photoabsorption is best described in relation to resonant scattering. Then the usual free-free rate (times an easily calculated coefficient) becomes, in effect, a contribution to the imaginary part of the self energy of the propagator for the resonant state of the electron (or proton), to be added to the imaginary part coming from the collisionless resonance decay. Of course, there is a real part to the self energy part as well, of the same order in the expansion parameter e^2 as the imaginary part. Since it is small we might have assumed that the real part is just an inconsequential shift of the resonance energy. However, it is energy dependent (and logarithmically singular at the position of the resonance). We find that including the real part greatly changes both the shape of the spectrum near the resonance and the integral over the resonance region.

b. To exhibit an approach to the "quantized proton" problem that we believe is superior to those found in the literature, and to present a result for the "Gaunt factor" that is much simpler than that contained in the appendex of ref. [5].

c. To address some collective effects that can be significant in domains of higher density.

In discussing the issues that we shall raise, it is much more efficient, even for the recapture of single electron results or classical results, to begin from a quantum field theory formulation. This formulation is at the same time well adapted for addressing the emission, absorption and scattering processes that are involved, and for incorporating the statistical mechanics of the plasma. We consider non-relativistic spinless electrons, of mass m annihilated by the field, $\psi_e(\mathbf{r})$, and protons of mass M annihilated by the field $\psi_p(\mathbf{r})$. We choose Coulomb gauge for the electromagnetic field. Denoting the vector potential for the external field $\mathbf{A}(\mathbf{r})$, we define the current operator, as $\mathbf{j}(\mathbf{r},t) = \mathbf{j}_e(\mathbf{r},t) + \mathbf{j}_p(\mathbf{r},t)$, where the currents for the respective fields {e,p} are,

$$\mathbf{j}_e = \frac{-e}{2m}\Big[\psi_e^\dagger[-i\vec{\nabla} - e\vec{A}]\psi_e + [i\vec{\nabla} - e\vec{A}]\psi_e^\dagger\ \psi_e\Big]\,,$$
$$\mathbf{j}_p = \frac{e}{2M}\Big[\psi_p^\dagger[-i\vec{\nabla} + e\vec{A}]\psi_p + [i\vec{\nabla} + e\vec{A}]\psi_p^\dagger\ \psi_p\Big]. \quad (1)$$

We also define the number density operators $n_{e,p}(\mathbf{r}) = \psi_{e,p}^\dagger(\mathbf{r})\psi_{e,p}(\mathbf{r})$. We divide the Hamiltonian into an unperturbed part H_0, which includes all interactions with the externally applied magnetic field,

$$H_0 = \frac{1}{2m}\int d^3r[i\vec{\nabla} + e\vec{A}(\mathbf{r})]\psi_e^\dagger \cdot [-i\vec{\nabla} + e\vec{A}(\mathbf{r})]\psi_e$$
$$+\frac{1}{2M}\int d^3r[i\vec{\nabla} - e\vec{A}(\mathbf{r})]\psi_p^\dagger \cdot [-i\vec{\nabla} - e\vec{A}(\mathbf{r})]\psi_p\,, \quad (2)$$

a Coulomb term, H_c, which we divide into two pieces $H_c = H_c^{(A)} + H_c^{(B)}$,

$$H_c^{(A)} = \frac{e^2}{8\pi}\Big[\int (d^3\mathbf{r})(d^3\mathbf{r}')n_e(\mathbf{r})\frac{1}{|\mathbf{r}-\mathbf{r}'|}n_e(\mathbf{r}')$$
$$+\int (d^3\mathbf{r})(d^3\mathbf{r}')n_p(\mathbf{r})\frac{1}{|\mathbf{r}-\mathbf{r}'|}n_p(\mathbf{r}')\Big]\,, \quad (3)$$

$$H_c^{(B)} = -\frac{e^2}{4\pi}\int (d^3\mathbf{r})(d^3\mathbf{r}')n_e(\mathbf{r})\frac{1}{|\mathbf{r}-\mathbf{r}'|}n_p(\mathbf{r}')\,, \quad (4)$$

and a radiation term, $H_{\rm rad}$, which couples the external radiation to the matter,

$$H_{\rm rad} = \int d^3r \Big\{ \big[\mathbf{j}_e + \mathbf{j}_p\big] \cdot \mathbf{A}_{\rm rad} + e^2 \Big[\frac{n_e}{2m} + \frac{n_p}{2M}\Big] \mathbf{A}_{\rm rad} \cdot \mathbf{A}_{\rm rad} \Big\} , \quad (5)$$

We choose the magnetic field $\mathbf{B}$ to be in the $\hat{z}$ direction. In this case it is convenient to define currents $j^{\pm}_{e,p} = ([j_{e,p}]_x \pm i[j_{e,p}]_y)/\sqrt{2}$ which couple photon polarization vectors of the form $(1, \mp i, 0)/\sqrt{2}$. In the dipole approximation to photon absorption and emission we shall encounter the space integrals of the currents (1)

$$\mathbf{J}_{e,p} = \int d^3r\, \mathbf{j}_{e,p}(\mathbf{r}) \, . \quad (6)$$

In a system governed by H_0 each particle in the ensemble of electrons and protons moves independently and the second quantized formalism is unnecessary. For a single electron system, for example, the action of the operator $\mathbf{J}_e$ on a state is the same as the action of the operator $e\Pi = e(\mathbf{p} - e\mathbf{A})$ on the wave function for that state. Thus the familiar relations for an electron in a constant magnetic field, $[\Pi^+_e, \Pi^-_e] = eB = m\omega_{ce}$, $[\Pi^{\pm}_e, H_0] = \pm\omega_{ce}\Pi^{\pm}_e$, and $[\Pi^+_p, \Pi^-_p] = -\omega_{cp}$, $[\Pi^{\pm}_p, H_0] = \mp\omega_{ce}\Pi^{\pm}_p$, translate into,

$$[J^{\pm}_e, H_0] = \mp\omega_{ce} J^{\pm}_e \quad , \quad [J^{\pm}_p, H_0] = \pm\omega_{cp} J^{\pm}_p \, . \quad (7)$$

where ω_{ce} and ω_{cp} are the respective electron and proton cyclotron frequencies. When we introduce the Heisenberg picture in the usual way, with $H = H_0 + H_c + H_{\rm rad}$, the equations (7) give,

$$\Big(\frac{i\partial}{\partial t} \pm \omega_{ce}\Big) J^{\pm}_e(t) = [J^{\pm}_e(t), (H_c(t) + H_{\rm rad}(t))] \, ,$$
$$\Big(\frac{i\partial}{\partial t} \mp \omega_{cp}\Big) J^{\pm}_p(t) = [J^{\pm}_p(t), (H_c(t) + H_{\rm rad}(t))] \, . \quad (8)$$

These are the key equations for our application. Turning to the commutators of the J's with the Coulomb Hamiltonian, it is more transparent if we use the 3D vector representation and express the $\mathbf{J}$'s in terms of infinitesimal translation operators for electron coordinates $\mathbf{T}_e$, and for proton coordinates, $\mathbf{T}_e$,

$$\mathbf{J}_e = \frac{-ie}{m}\mathbf{T_e} + e^2 \int d^3r\, n_e(\mathbf{r}) \mathbf{A}(\mathbf{r}) \, ,$$
$$\mathbf{J}_p = \frac{ie}{M}\mathbf{T_p} + e^2 \int d^3r\, n_p(\mathbf{r}) \mathbf{A}(\mathbf{r}), \quad (9)$$

where,

$$[\mathbf{T}_{e,p}, \psi_{e,p}(\mathbf{r})] = \nabla \psi_{e,p}(\mathbf{r}) \, . \quad (10)$$

Noting that $[n_{e,p}(\mathbf{r})\mathbf{A}(\mathbf{r}), H_c] = 0$ we find first that,

$$[\mathbf{J}_{e,p}, H^{(A)}_c] = 0 \, , \quad (11)$$

since the e-e and p-p interaction terms are separately invariant under the separate translations of either the electron or proton coordinate. However the e-p interaction $H^{(B)}_c$ is invariant only under simultaneous translations for the electrons and protons,

$$[\mathbf{T}_e + \mathbf{T}_p, H_c] = \Big[\Big(\frac{m}{ie}\mathbf{J}_e - \frac{M}{ie}\mathbf{J_p}\Big), H_c\Big] = 0 \, . \quad (12)$$

The separate commutators needed in (8)are thus,

$$[\mathbf{J_e}, H_c] = -\frac{M}{m}[\mathbf{J_p}, H_c]$$
$$= i\frac{e^2}{4\pi m} \int (d^3\mathbf{r})(d^3\mathbf{r}')\, \nabla_{\mathbf{r}} n_e(\mathbf{r}, t) \frac{1}{|\mathbf{r} - \mathbf{r}'|} n_p(\mathbf{r}', t) \, . \quad (13)$$

FORMAL EXPRESSIONS FOR THE PHOTO-ABSORPTION RATE.

Since the issues raised in the present paper pertain largely to the modes polarized perpendicularly to the magnetic field, we address only the absorption of these modes. We begin from the expression that the field theoretic formulation of statistical mechanics provides for the photo-absorption rate $\gamma_A(\omega)\pm$ in the medium, where $\pm$ refers to the photon modes with polarization vector $(1, \pm i, 0)\sqrt{2}$

$$\gamma_A(\omega)^{\pm} = \tfrac{1}{2\omega[Vol.]} \int d^3x\, d^3y\, dt\, e^{i\omega t} e^{-i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y})} \times \langle j^{\pm}(\mathbf{x}, t) j^{\mp}(\mathbf{y}, 0)\rangle \, . \quad (14)$$

The Heisenberg operators $j^{\pm}(x,t)$ are the relevant combinations of the total electromagnetic current operator, $j^{\pm} = ([j_e + j_p]_1 \pm i[j_e + j_p]_2)/\sqrt{2}$. The brackets, $\langle\rangle$ stand for thermal average, $\langle A\rangle = {\rm Tr}[{\rm e}^{-\beta {\rm H}} {\rm A}]/{\rm Tr}[{\rm e}^{-\beta {\rm H}}]$ where $\beta = T^{-1}$. Taking the dipole limit, $q \to 0$, and using the definition $J^{\pm}_{e,p} = \int d^3r j^{\pm}_{e,p}(\mathbf{r})$ we obtain, after an integration by parts,

$$\gamma_A(\omega)^{\pm} = \tfrac{1}{2\omega} \int dt\; e^{i\omega t} \tfrac{1}{\omega \pm \omega_{ce}} \Big(\tfrac{i\partial}{\partial t} \pm \omega_{ce}\Big) \langle J^{\pm}_e(t)\, j^{\mp}(\mathbf{0}, 0)\rangle + \tfrac{1}{2\omega} \int dt\; e^{i\omega t} \tfrac{1}{\omega \mp \omega_{cp}} \Big(\tfrac{i\partial}{\partial t} \mp \omega_{cp}\Big) \langle J^{\pm}_p(t)\, j^{\mp}(\mathbf{0}, 0)\rangle \, . \quad (15)$$

Now we substitute (8), letting $A^{\rm rad} = 0$ since (at the moment) we are considering a process in which no photons are involved except the one being absorbed.

$$\gamma_A(\omega)^{\pm} = \tfrac{1}{2\omega} \int dt\; e^{i\omega t} \langle \Big[\Big\{ \tfrac{J^{\pm}_p(t)}{\omega \mp \omega_{cp}} + \tfrac{J^{\pm}_e(t)}{\omega \pm \omega_{ce}} \Big\}, H_c(t) \Big] j^{\mp}(0)\rangle$$
$$= \tfrac{-1}{2\omega} \int dt\; e^{i\omega t} \langle \Big[\Big\{ \tfrac{j^{\mp}_p(\mathbf{0},0)}{\omega \mp \omega_{cp}} + \tfrac{j^{\mp}_e(\mathbf{0},0)}{\omega \pm \omega_{ce}} \Big\}, H_c(0) \Big] J^{\pm}(t)\rangle \, . \quad (16)$$

The second form follows from translational invariance (to shift the space integral from the first to the second current), space inversion (introducing a (-) sign) and

time translational invariance (a displacement $-t$) in the thermal average factor, followed by complex conjugation. Performing the same steps once again we obtain,

$$\gamma_A(\omega)^{\pm} = \tfrac{-1}{2\omega}\int dt\; e^{i\omega t}\langle\Big[\Big(\tfrac{J_p^{\mp}(t)}{\omega\mp\omega_{cp}}+\tfrac{J_e^{\mp}(t)}{\omega\pm\omega_{ce}}\Big),H_c(t)\Big] \times\Big[\Big(\tfrac{j_p^{\pm}(\mathbf{0},0)}{\omega\mp\omega_{cp}}+\tfrac{j_e^{\pm}(\mathbf{0},0)}{\omega\pm\omega_{ce}}\Big),H_c(0)\Big]\rangle\,. \tag{17}$$

Using (9) and (11) we can rewrite (17)in a simpler form[1]

$$\gamma_A(\omega)^{\pm} = \frac{1}{2\omega[\mathrm{Vol.}]}\Big(\frac{1}{M(\omega\pm\omega_{cp})}+\frac{1}{m(\omega\mp\omega_{ce})}\Big)^2 \times\int dt\; e^{i\omega t}\langle\Big[T_e^{\pm}(t),H_c(t)\Big]\Big[T_e^{\mp}(0),H_c(0)\Big]\rangle\,. \tag{18}$$

or explicitly, using (12),

$$\gamma_A(\omega)^{\pm} = \tfrac{e^6}{32\pi^2\omega}\Big[\tfrac{\omega}{m(\omega\pm\omega_{cp})(\omega\mp\omega_{ce})}\Big]^2 \times\int(d^3\mathbf{r}_1)(d^3\mathbf{r}_2)(d^3\mathbf{r}_3)\,\tfrac{1}{|\mathbf{r}_1-\mathbf{r}_2|}\,\tfrac{1}{|\mathbf{r}_3|}\int dt\, e^{i\omega t} \times\langle\partial_{\pm}n_e(\mathbf{r}_1,t)n_p(\mathbf{r}_2,t)\partial_{\mp}n_e(\mathbf{r}_3,0)n_p(0,0)\rangle\;. \tag{19}$$

where we have defined, $\partial_{\pm}\equiv(\partial/\partial x\pm i\partial/\partial y)/\sqrt{2}$, and simplified the prefactor using $M\omega_{cp}=m\omega_{ce}$ and $(m+M)/M\approx 1$. This prefactor in (19) agrees with that found in ref. [5], except for the damping terms that matter only very near the resonance in the results of these authors. This subject will be discussed at length in secs. 4 and 5.

To recapitulate, (19) gives the exact rates in the dipole limit. All of the effects of Coulomb interactions in the medium are included. Of course, the four-density correlator must calculated in an approximation. There are different domains in which it makes sense to take different approaches. For example, at high temperatures and low densities we can use the result in the absence of Coulomb coupling (beyond that already exhibited explicitly in (19)). For low temperatures and low densities we need to include full Coulomb wave-functions in the electron's interactions with a single ion. For high densities collective effects become interesting, and "ring approximation" sums required (at the least).

BORN APPROXIMATION IN TWO REGIONS

The four-point density correlator depends on both Coulomb and magnetic interactions. By "Born approximation" we mean that the Coulomb couplings are turned off in calculating the bracket in (19). In this case the correlator factors into an electron part and a proton part, separately translationally invariant,

$$\langle\partial_{\pm}n_e(\mathbf{r}_1,t)n_p(\mathbf{r}_2,t)\partial_{\mp}n_e(\mathbf{r}_3,0)n_p(\mathbf{0},0)\rangle = \langle\partial_{\pm}n_e(\mathbf{r}_1,t)\partial_{\mp}n_e(\mathbf{r}_3,0)\rangle\langle n_p(\mathbf{r}_2,t)n_p(\mathbf{0},0)\rangle\,. \tag{20}$$

Using (41) and taking Fourier transforms in (19) gives

$$\gamma_A(\omega)^{\pm} = \frac{1}{\omega\pi}\Big(\frac{e^2}{4\pi}\Big)^3\Big[\frac{\omega}{m(\omega\pm\omega_{cp})(\omega\mp\omega_{ce})}\Big]^2F(\omega)\,, \tag{21}$$

where,

$$F(\omega)=\int d\omega_1\,d^3k\Big[\frac{k_\perp^2}{(k^2)^2}\Big]\Big[\Delta_e(\mathbf{k},\omega-\omega_1)\Big]\Big[\Delta_p(\mathbf{k},\omega_1)\Big]\,, \tag{22}$$

and

$$\Delta_{e,p}(\mathbf{k},\omega)=\int d^4x\,e^{i\mathbf{k}\cdot\mathbf{x}}e^{i\omega t}\langle n_{e,p}(\mathbf{x},t)n_{e,p}(\mathbf{0},0)\rangle\,. \tag{23}$$

to be evaluated in the absence of Coulomb interactions. We work out (22) in two domains; $\omega_{cp}<<T<1.5\omega_{ce}$ and $T<<1.5\omega_{cp}$, the first being a domain in which the electron occupancy is mainly confined to the ground and first excited Landau levels, with the magnetic effects on the proton being negligible. The second domain is one in which the electrons are strongly confined to the lowest level and the protons are mainly confined to the ground and first excited Landau levels.

Region: $\omega_{cp}<<T<1.5\omega_{ce}$

In this domain the free proton correlator is simply,

$$\Delta_p(\mathbf{k},\omega)=2\pi n_e^{(0)}\delta(\omega)\,. \tag{24}$$

There are collective effects that modify this correlator to which we return later. We introduce the variables,

$$\xi_0=\sqrt{\tfrac{m\beta}{2}}\Big(\tfrac{\omega}{k_\parallel}-\tfrac{k_\parallel}{2m}\Big)\quad,\quad \xi_1=\sqrt{\tfrac{m\beta}{2}}\Big(\tfrac{\omega-\omega_{ce}}{k_\parallel}-\tfrac{k_\parallel}{2m}\Big)\,, \quad \xi_{-1}=\sqrt{\tfrac{m\beta}{2}}\Big(\tfrac{\omega+\omega_{ce}}{k_\parallel}-\tfrac{k_\parallel}{2m}\Big)\quad,\quad \zeta=\tfrac{k_\perp^2}{2m\omega_{ce}} \tag{25}$$

Following the rules given in the appendix we have calculated the terms in the electron correlator that come from the first two Landau levels only,

$$\Delta_e(k_\parallel,k_\perp,\omega)=(1-e^{-\beta\omega_{ce}})\sqrt{2\pi m\beta}\,n_e^{(0)}|k_\parallel|^{-1}e^{-\zeta} \times\Big[\zeta\exp(-\xi_1^2)+[1+(1-\zeta)^2e^{-\beta\omega_{ce}}]\exp(-\xi_0^2) +e^{-\beta\omega_{ce}}\zeta\exp(-\xi_{-1}^2)\Big]\,. \tag{26}$$

[1] The volume, Vol. , enters or leaves our formulae depending on whether or not we use, e.g., the current evaluated at an arbitrary point in space, $j_e^{\mp}(0,0)$, as in (17) or the space integral of the current which, as it occurs in (18), is just the current at the point times the volume, in view of the translational invariance of the thermal average factor in (17).

Substituting (26) and (24) into (22), introducing a variable $s = k_\perp^2/k_\parallel^2$, and then doing the $k_\parallel$ integration, we obtain,

$$F(\omega) = \frac{16}{3}\sqrt{2m\beta}\,\pi^{3/2}[n_e^{(0)}]^2\Lambda\,, \tag{27}$$

where Λ has exactly the same meaning as in ref. [5], and is given by,

$$\Lambda = \tfrac{3}{4}(1-e^{-\beta\omega_{ce}})e^{\beta\omega/2}\int_0^\infty ds\tfrac{s}{(1+s)^2}$$
$$\times\Big[K_0\Big(\beta\omega\sqrt{.25+s/\omega_{ce}\beta}\Big)\Big(1+\tfrac{3-2s+s^2}{(s+1)^2}e^{-\beta\omega_{ce}}\Big)$$
$$+(1+s)^{-1}e^{-\beta\omega_{ce}/2}\Big[K_0\Big(\beta|\omega-\omega_{ce}|\sqrt{.25+s/\omega_{ce}\beta}\Big)$$
$$+K_0\Big(\beta|\omega+\omega_{ce}|\sqrt{.25+s/\omega_{ce}\beta}\Big)\Big]\Big]\,. \tag{28}$$

This answer is identical to the results of Pavlov and Panov [1], as corrected by Potekhin and Chabrier in eqn.(44) of ref. [5], when the latter is expanded in powers of $\exp[-\beta\omega_{ce}]$, and only the zeroth and first order terms retained.

Region: $T < 1.5\omega_{cp}$

We choose the region to extend to $1.5\omega_{cp}$ in order to capture the resonance behavior, while keeping the electrons strongly confined to the lowest Landau level. Therefore we take only the term with unity in the final factor in (26) for the electronic correlator.. The contribution of the first two Landau levels to the proton correlator is given by taking $m \to M$, and $\omega_{ce} \to \omega_{cp}$ in (26) as it stands. Doing the ω_1 integral in (21), discarding terms of relative order m/M, and setting $\exp(-\beta\omega_{ce}) = 0$, appropriate to the temperature regime, we obtain,

$$F(\omega) = (1-e^{-\beta\omega_{cp}})\sqrt{2m\beta}\,\pi[n_e^{(0)}]^2\int\tfrac{d^3k}{|k_\parallel|}\Big[\tfrac{k_\perp^2}{(k_\parallel^2+k_\perp^2)^2}\Big]$$
$$\times e^{-2\zeta}\exp\Big[\tfrac{\beta\omega}{2}-\tfrac{k_\parallel^2\beta}{8m}\Big]\Big\{\Big(1+e^{-\beta\omega_{cp}}(1-\zeta)^2\Big)$$
$$\times\exp\Big[-\tfrac{\omega^2 m\beta}{2k_\parallel^2}\Big]+\zeta e^{-\beta\omega_{cp}/2}\exp\Big[-\tfrac{(\omega-\omega_{cp})^2 m\beta}{2k_\parallel^2}\Big]$$
$$+\zeta e^{-\beta\omega_{cp}/2}\exp\Big[-\tfrac{(\omega+\omega_{cp})^2 m\beta}{2k_\parallel^2}\Big]\Big\}\,. \tag{29}$$

In the calculation we replaced the reduced mass by m in several places. Then the only place the proton mass enters is through ω_{cp}. Note that $\zeta = k_\perp^2/(m\omega_{ce}) = k_\perp^2/(M\omega_{cp})$. Doing the $k_\parallel$ integration we obtain (27) with Λ replaced by Λ', where

$$\Lambda' = \tfrac{3}{4}(1-e^{-\beta\omega_{cp}})e^{\beta\omega/2}\int_0^\infty ds\tfrac{s}{(1+s)^2}$$
$$\times\Big[K_0\Big(\beta\omega\sqrt{.25+2s/\omega_{ce}\beta}\Big)\Big(1+\tfrac{2s^2+1}{2(s+1)^2}e^{-\beta\omega_{cp}}\Big)$$
$$+(1+s)^{-1}e^{-\beta\omega_{cp}/2}K_0\Big(\beta|\omega-\omega_{cp}|\sqrt{.25+2s/\omega_{ce}\beta}\Big)$$
$$+(1+s)^{-1}e^{-\beta\omega_{cp}/2}K_0\Big(\beta|\omega+\omega_{cp}|\sqrt{.25+2s/\omega_{ce}\beta}\Big)\Big]\,. \tag{30}$$

Note the very close resemblance to (28), even though (30) is to be used in a domain of temperature 1000 times smaller, at a given magnetic field. Of course when we took the two lowest Landau states for the case of quantized protons, rather than for electrons, the energy difference ω_{ce} in (28) is replaced by ω_{cp} in (30). But note that it is still the electronic parameter ω_{ce} that enters the $\sqrt{.25+2s/\omega_{ce}}$ factor in the arguments of the Bessel functions, , but with a coefficient that is different by a factor of 2. When the temperature is so low that we have $\omega_{ce}\beta = 2000$, then the integral of the first K_0 function in (30) becomes rather large, since convergence for large s comes from the cutoff supplied by the K_0 function.

Potekhin and Chabrier [5] have given formulae which should exactly agree with (30). However, they went quite a different route to obtain these formulae and end up with complex expressions that we have not been able to cast into our form. However, we obtain a plot similar that of the lower panel in their fig. 6. showing the peak in the scattering rate for the anti-resonant polarization, when plotted over the proton resonance region.

ELECTRON RESONANCE REGION.

We now address aspects of behavior in the resonant region that we believe are not adequately treated in the present literature. Specializing to the case of our first domain $\omega_{cp} << T < 1.5\omega_{ce}$ appropriate to the electron resonance region, (21) becomes

$$\gamma_A(\omega) = \frac{e^6}{64\pi^4\omega}\Big[\frac{1}{m(\omega-\omega_{ce})}\Big]^2 F(\omega)\,, \tag{31}$$

an expression that doesn't exist at the resonance frequency; we must turn to the $\omega \to \omega - i\epsilon$ prescription for the definition. The way that this works is that we first recognize (31) for the non-resonant case as the imaginary part of an amplitude defined by the graph of fig 2.

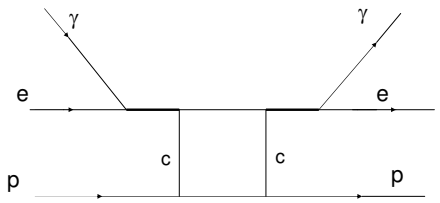

FIG. 1: The basic $\gamma + e \to \gamma + e$ graph, the imaginary part of which gives the resonant part of the photoabsorption rate. The heavy lines indicate the first excited Landau level.

Accordingly, we replace (31) by

$$\gamma_A(\omega) = \frac{e^6}{64\pi^4\omega}\mathrm{Im}\Big[\Big(\frac{1}{m(\omega-\omega_{ce})}\Big)^2 G(\omega - i\epsilon)\Big]\,. \quad (32)$$

where $F(\omega) = \mathrm{Im}G(\omega - i\epsilon)$. When we move into the resonance region the denominator factor $(\omega - \omega_{ce})$ must have an imaginary part as well. In our original definition of the problem in which the only photons are the external ones that coupled to the currents in (14) this imaginary part originates in a self energy insertion, $\Sigma^{(ff)}$ in the inverse propagator of the resonance. The imaginary part, $\mathrm{Im}\Sigma^{(ff)}$ comes from our earlier calculation, but redescribed (with the appropriate multiplying coefficients) as the collisional deexcitation rate, ν_{ff} for the resonant state. Of course, the resonance inverse propagator also has an imaginary part coming from the intermediate state in which a photon reappears, defining the radiative (or natural) width $\nu_{re} = (2/3)e^2\omega_{ce}\omega m^{-1}$ in the absence of Coulomb collisions. We denote the sum of the two imaginary parts, $\mathrm{Im}[\Sigma^{(ff)} + \Sigma^{(r)}]$, by ν_e, where $\nu_e = \nu_{ff} + \nu_{re}$.

It is conceptually incorrect simply to replace $(\omega - \omega_{ce})^{-2}$ in (32) by $[(\omega - \omega_{ce})^2 + \nu_e^2]^{-1}$. It also can lead to very incorrect numerical results, depending on the magnitude of $\mathrm{Re}[G]$. Instead (32) should be replaced by

$$\gamma_A(\omega) = \frac{e^2 n_e^{(0)}\omega_{ce}}{\omega m}\mathrm{Im}[\omega - \omega_{ce} - \Sigma^{(r)} - \Sigma^{(ff)}]^{-1}\,, \quad (33)$$

where $\nu_e = \mathrm{Im}[\Sigma^{(r)} + \Sigma^{(ff)}]$. The prefactor in (33) is determined by expanding to first order in $\mathrm{Im}\Sigma^{(ff)} = \nu_{ff}$ and comparing to (31), with the identification,

$$\nu_{ff} = \frac{e^4\mathrm{Im}\Sigma^{(ff)}(\omega)}{64\pi^4 m\omega_{ce}n_e^{(0)}} = \frac{\alpha^2 F(\omega)}{4\pi^2 m\omega_{ce}n_e^{(0)}}\,. \quad (34)$$

A graphical representation of (33) is shown in fig. 2.

The result (33), where we use $\Sigma^{(r)} + \Sigma^{(ff)} = i(\nu_{ff} + \nu_{re})$, gives exactly the usual results [5] for the resonant region. Now, however, we can ask about the effects of $\Sigma^{(ff)}$ and $\Sigma^{(r)}$ If these real parts of are small and not too energy dependent, then they provide small shifts in the resonance energy that make no difference either to the total rate or to the shape of the spectrum. But it turns out that $\Sigma^{(ff)}(\omega)$ has a logarithmic singularity in its real part at $\omega = \omega_{ce}$ that is of the same nature as that in its imaginary part, and we will see that it can no longer be dismissed.

In calculating $\mathrm{Re}\Sigma^{(ff)}$, we can avoid introducing a formalism with propagators for the resonant state by directly writing

$$\Sigma^{(ff)} = \frac{\alpha^2}{\pi m\omega_{ce}}\int d^3k\Big[\frac{k_\perp^2}{(k^2)^2}\Big]\Big[\frac{[\Pi_e(\mathbf{k},\omega)]}{(1-e^{-\beta\omega})}\Big], \quad (35)$$

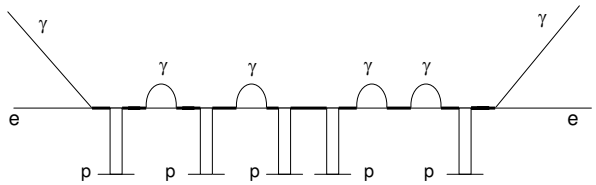

FIG. 2: A typical graph that enters the photoabsorption calculation in the resonance region. The incoming photon is absorbed on an electron in the lowest Landau level (exciting the electron to the next Landau level). The initial state is restored at the right hand side of the diagram. In between we have the propagator for the resonance, as given in (34), with self energy insertions corresponding both to the "free-free" scattering from protons and from free decay into photons as well. The photoabsorption rate is found from the imaginary part of the sum of all such graphs.

where we have used (22) and (24), and have extended $\Delta(\mathbf{k},\omega)$ of (22) to the complex plane with,

$$\Delta(\mathbf{k},\omega) = \frac{\mathrm{Im}[\Pi_e(\mathbf{k},\omega)]}{(1-e^{-\beta\omega})}\,, \quad (36)$$

The imaginary part of (35) reproduces the above results for ν_{ff}. The function $\Pi_e(\mathbf{k},\omega)$ is the Fourier transform of the retarded commutator (see ref.[12], sec. 33),

$$\Pi_e(\mathbf{r},t) = \langle[n_e(\mathbf{r},t), n_e(0,0)]\rangle\theta(t)\,. \quad (37)$$

which supplies the extension to the complex plane that has the correct analytic properties.

To construct the real part of Π we extend the variable set (25),

$$\xi_0^\pm = \sqrt{\tfrac{m\beta}{2}}\Big(\tfrac{\omega}{k_\parallel} \pm \tfrac{k_\parallel}{2m}\Big)\ ,\ \ \xi_1^\pm = \sqrt{\tfrac{m\beta}{2}}\Big(\tfrac{\omega-\omega_{ce}}{k_\parallel} \pm \tfrac{k_\parallel}{2m}\Big)\,,$$
$$\xi_2^\pm = \sqrt{\tfrac{m\beta}{2}}\Big(\tfrac{\omega+\omega_{ce}}{k_\parallel} \pm \tfrac{k_\parallel}{2m}\Big)\ ,\ \ \zeta = \tfrac{k_\perp^2}{2m\omega_{ce}}\,, \quad (38)$$

and we define the usual plasma function,

$$\Phi(\xi) = 2e^{-\xi^2}\int_0^\xi dy\, e^{y^2}\,. \quad (39)$$

Then we find,

$$\mathrm{Re}[\Pi_e(\mathbf{k},\omega)] = \tfrac{\sqrt{m\beta}\,n_e e^{-\zeta}(1-e^{-\beta\omega_{ce}})}{\sqrt{2}k_\parallel}$$
$$\times\Big[\Big(\Phi(\xi_0^-) - \Phi(\xi_0^+)\Big)\Big(1 + e^{-\beta\omega_{ce}}(1-\zeta)^2\Big)$$
$$+\zeta\Big(\Phi(\xi_1^-) - \Phi(\xi_2^+)\Big) + \zeta e^{-\beta\omega_{ce}}\Big(\Phi(\xi_2^-) - \Phi(\xi_1^+)\Big)\Big]\,. \quad (40)$$

With the real part of $\Sigma^{(ff)}$ of (35) determined from (40) and the imaginary part from (36) the absorption rate is calculated from from (33). In fig. 3 we show the difference that inclusion of the real part of the resonance self energy term can make in a region near the resonance peak., for the case of parameters, $\rho = 10gc^{-3}$, $\omega_{ec} = 1$ KeV,$T = .2$KeV. Plotted are the opacities in a region $1.01\text{KeV} < \omega < 1.1\text{KeV}$, coming from (33) using the real part of $\Sigma^{(e)}$ that comes from (40) and for the imaginary part simply ν_{ff} as computed in section 3. Neither the real nor the imaginary parts of $\Sigma^{(re)}$ make a significant contribution in this region.

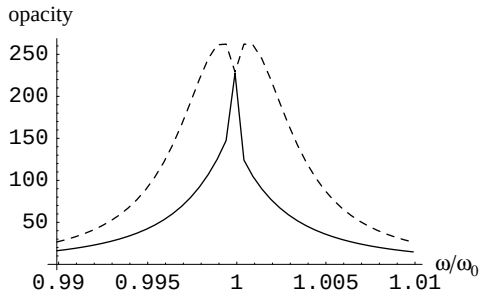

FIG. 3: Comparison of the absorption rates, γ_A, very near the electron resonance with and without inclusion of the real part of the self energy part in the resonance propagator. The dashed curve is the conventional model with no real part. The resonance energy is .2 KeV and the plot is from 1% below the resonance energy to one percent above.

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PROTON RESONANCE REGION

With these results of the previous section in mind, we return to the case in which both proton and electron currents contribute, where the interference term is important. We will consider the $^{(+)}$ polarization state, which resonates with the proton at energy ω_{cp}, but never with the electron, and stay in the energy region $\omega << \omega_{ce}$ throughout. We go back to (21), rewritten to separate the proton and electron current as in (18),

$$\gamma_A(\omega) = \frac{e^6 F(\omega)}{\omega(4\pi)^3}\Big(\frac{1}{M(\omega-\omega_{cp})} + \frac{1}{m(\omega+\omega_{ce})}\Big)^2$$
$$\approx \frac{e^6 F(\omega)}{\omega M^2(4\pi)^3}\Big[\omega_{cp}^{-2} + 2\omega_{cp}^{-1}(\omega-\omega_{cp})^{-1} + (\omega-\omega_{cp})^{-2}\Big]\,, \tag{41}$$

where we have used $m\omega_{ec} = M\omega_{pc}$, and $\omega << \omega_{ce}$ to get the second form.

In the first term the photon is absorbed on the electron; since we are totally out of the resonance region we do not need to build it into the propagator for the electron resonance, as above, although there would be no harm in doing so. We evaluate it as is, denoting the contribution as $\gamma_{(1)}$,

$$\gamma_{(1)} = \frac{e^6 F(\omega)}{\pi(4\pi)^3\omega M^2}\omega_{cp}^{-2}\,. \tag{42}$$

The cross term, that is, the interference between the amplitude in which a photon is absorbed on the electron and that in which the photon is absorbed on the proton, is trickier. It is not a constant times the imaginary part of a self-energy in a resonance propagator; we evaluate it almost as it stands. We recognize that it is a piece of an imaginary part of a function of something with four legs, but save ourselves the chore of further formal definitions, by noting that the function $F(\omega)$ defined in (22) is the imaginary part of an analytic function, $G(\omega)$, the real part of which we have calculated, but which is insignificant in what follows. The contribution of the second (cross) term in (41) becomes,

$$\gamma_{(2)}(\omega) = \frac{2e^6}{(4\pi)^3\omega\omega_{cp}M^2}\mathrm{Im}[\frac{G(\omega)}{\omega-\omega_{cp}-i\nu_p(\omega)}]$$
$$\approx \frac{2e^6}{(4\pi)^3\omega\omega_{cp}M^2}\frac{(\omega-\omega_{cp})F(\omega)}{(\omega-\omega_{cp})^2+\nu_p(\omega)^2}\,, \tag{43}$$

where ν is just the imaginary part of the proton-resonance inverse propagator under the influence both of the magnetic field and the Coulomb scattering, $\nu_p = \nu_{rp} + \nu_{ff}$ with $\nu_{rp} = (2/3)e^2M^{-1}\omega\omega_{cp}$. In the second line of (43) we have set the real part of G equal to zero after all, after calculating it approximately and finding it inconsequential. (It was important to have it included in principle, since our calculation should always be that of calculating the imaginary part of an analytic function.)

As we found in the last section the free-free width term in the denominator ν_{ff} can be expressed in terms of the function F of (22)as

$$\nu_{ff}(\omega) = \frac{\alpha^2 F(\omega)}{4\pi^2 m\omega_{ce}n_e^{(0)}}\,, \tag{44}$$

where for this case $F(\omega)$ is evaluated as

$$F(\omega) = \frac{16}{3}\sqrt{2m\beta}\,\pi^{3/2}[n_e^{(0)}]^2\Lambda'\,, \tag{45}$$

where Λ' is evaluated in (30).

The third term, in (41) leads to a contribution $\gamma_A(\omega)_{(3)}$ where the factor $(\omega-\omega_{cp})^{-2}$ needs to be fitted into the propagator for the proton resonant state, exactly in the fashion used in the pure electron problem to obtain (33)

$$\gamma_{(3)} = \frac{e^2 n_e^{(0)} \omega_{cp}}{\omega M} \mathrm{Im}[\omega - \omega_{cp} - i\nu_{rp} - i\nu_{ff}]^{-1} . \quad (46)$$

We can now add the three contributions, replacing (41) by, $\gamma_A \to \gamma_{(1)} + \gamma_{(2)} + \gamma_{(3)}$,

$$\gamma_A(\omega) = \frac{e^2 n_e^{(0)}}{\omega \omega_{cp} M} \Big[\frac{\nu_{ff}\omega^2 + \nu_{rp}\omega_{cp}^2}{(\omega - \omega_{cp})^2 + \nu_p^2)} \Big] . \quad (47)$$

Again we appear to disagree with the results of ref. [5]. According to eq. 53 of that paper, the free-free part of the damping term corresponding to ν_p as it occurs in the denominator of (47), above, carries an additional factor of m/M compared to our expression. Also the second term in the numerator of (47) is missing in ref. [5].

COLLECTIVE EFFECTS

For problems involving Coulomb forces in a plasma, screening is the leading collective effect. Indeed the authors of ref. [7], and previous authors as well, in effect replace the factor $[k^2]^{-2}$ in (22) by the Fourier transform of a screened potential, $[k^2 + \kappa_D^2]^{-2}$, where κ_D is the usual screening parameter. This replacement is already known to be incorrect in the $B = 0$ case; the correct static screening correction for photo-absorption in a hydrogen plasma has been shown [14] instead to be the replacement,

$$k^{-4} \to k^{-2} \frac{1 + \kappa_D^2/(2k^2)}{k^2 + \kappa_D^2} . \quad (48)$$

We expect the same result for the magnetic case; in accord with Sitenko's conclusions [15], but we shall nonetheless look at the matter in some detail, as there are non-static corrections that may be significant in some regions. We return to (22) but re-express the right hand side using (37) to obtain,

$$F(\omega) = \int d\omega_1 \, d^3k \Big[\frac{k_\perp^2}{k^4} \Big] \frac{\mathrm{Im}\Big[\Pi_e'(\mathbf{k}, \omega - \omega_1)\Big]}{(1 - e^{-\beta(\omega - \omega_1)})} \frac{\mathrm{Im}\Big[\Pi_p'(\mathbf{k}, \omega_1)\Big]}{(1 - e^{-\beta\omega_1})} , \quad (49)$$

where the Π' are the functions defined by (37), but now calculated, in approximation, in the presence of Coulomb forces, whereas the the functions, Π are the functions defined in (37)in the absence of Coulomb forces. In the ring approximation, we have,

$$\Pi_e'(\mathbf{k}, \omega) = \frac{[k^2 + 4\pi e^2 \Pi_p(\mathbf{k}, \omega)]\Pi_e(\mathbf{k}, \omega)}{k^2 + 4\pi e^2 \Pi_e(\mathbf{k}, \omega) + 4\pi e^2 \Pi_p(\mathbf{k}, \omega)} ,$$
$$\Pi_p'(\mathbf{k}, \omega) = \frac{[k^2 + 4\pi e^2 \Pi_e(\mathbf{k}, \omega)]\Pi_p(\mathbf{k}, \omega)}{k^2 + 4\pi e^2 \Pi_p(\mathbf{k}, \omega) + 4\pi e^2 \Pi_e(\mathbf{k}, \omega)} . \quad (50)$$

We can we describe this construction as the expression of the complete polarization parts Π_e', Π_p' from the proper polarization parts, Π_e, Π_p, then setting the proper parts to their values in the absence of Coulomb interactions. The best systematic derivation that we know of for these relations in a multicomponent classical plasma is in ref. [13], eq. 2.110.

The relations hold in the presence of quantum effects as well. We have obtained tractable expressions from substituting the results of (50) into (49) only for the first case of section 3, $\omega_{cp} << T < 1.5\omega_{ce}$, where we can neglect the magnetic interactions of the protons. For this case the large proton mass leads to the imaginary part of the proton correlator $\mathrm{Im}[D_p]$ in (21) being concentrated at very small values of $\omega_1 << T$, for relevant values of $k \approx \sqrt{mT}$. Thus in (21) we can set $\omega - \omega_1 = \omega$, and $(1 - \exp[-\beta\omega_1])^{-1} = (\beta\omega_1)^{-1}$, and use the dispersion relation to do the ω_1 integral,

$$\pi^{-1}\beta^{-1} \int d\omega_1 \mathrm{Im}\Pi_p'(\mathbf{k}, \omega_1)/\omega_1 = \beta^{-1}\mathrm{Re}\Pi_p'(\mathbf{k}, 0)$$
$$= \frac{4\pi e^2 n_e^{(0)}(1 + \kappa_e^2/k^2)}{(1 + \kappa_p^2/k^2 + \kappa_e^2/k^2)} , \quad (51)$$

where we have simplified by setting

$$4\pi e^2 \Pi_{e,p}(\mathbf{k}, 0) \approx 4\pi e^2 \Pi_{e,p}(0, 0) = \kappa_{e,p}^2 . \quad (52)$$

Here the $\kappa_{e,p}^2$ are the contributions of the individual species to the squared Debye wave number $\kappa_D^2 = \kappa_e^2 + \kappa_p^2$ and $\kappa_p^2 = \kappa_e^2 = 4\pi\beta e^2 n_e^{(0)}$. We have also rewritten the multiplying factor of κ_p^2 in (43) terms of the average proton density, $n_p^{(0)} = n_e^{(0)}$ and the temperature, β^{-1}. For the static response function ($\omega = 0$) this is a good approximation for all cases under consideration.

Putting these steps into (21), using the ring approximation (50) for the electron polarization, and noting that $\Pi_p(k, \omega \approx \beta^{-1}) \approx 0$ gives

$$F(\omega) = \frac{n_e^{(0)}}{1 - e^{-\beta\omega}} \int d^3k \Big[\frac{k_\perp^2}{k^4} \Big] \frac{\mathrm{Im}\Big[\Pi_e^{(0)}(\mathbf{k}, \omega)\Big]}{|1 + \Pi_e^{(0)}(\mathbf{k}, \omega)/k^2|^2}$$
$$\times \frac{(1 + \kappa_e^2/k^2)}{(1 + \kappa_p^2/k^2 + \kappa_e^2/k^2)} . \quad (53)$$

We can compare (53) for the case $B = 0$ with results in the literature giving the effects of Coulomb correlations on the photo-absorption rate. Taking the appropriate limits of the prefactor in (19), substituting the $B = 0$ form for $\mathrm{Im}\Pi_e^{(0)}/k^2$, and defining $\epsilon(k, \omega) = 1 + \Pi_e^{(0)}(k, \omega)/k^2$, and including the longitudinally polarized modes gives back exactly eq. (3) in the paper by Iglesias and Rose [16], and essentially the results of ref. [17] as well.[2] The corrections from the $|\epsilon|^2$ in the

[2] There is one discrepancy; in both of these references the dielec-

denominator and from the ionic correlator are actually relatively small in domains of density and temperature in which the plasma is weakly coupled, that is to say, in regions in which we can calculate at all. We note that the last factor on the right hand side of (53) provides the screening factor that we quoted at the beginning of this section.

DISCUSSION

We summarize our differences from previous authors in three different parameter regions:

1. Region of non-quantized protons. Here we recapture the results of other authors for the "logarithm", or "Gaunt factor", Λ. But there is another effect, at the same level of approximation, that can change the shape of the resonant peak by a lot, namely the rapid energy dependence of the real part of the self energy in the propagator for the resonance. Because of this energy dependence the real part is not merely a small adjustment to the resonant energy parameter, and can make large modifications to both the spectrum shape and the integral over the resonance region. We believe that the effect will be important in the proton-resonance region as well, but we have not calculated examples.

2. Region very close to the proton resonance. While we agree with the authors of ref. [5] in the way that the free resonance decay partial width parameter ν_{rp} enters the total width parameter ν_p, we appear to disagree with these authors on the contribution of the free-free parameter ν_{ff} itself to this width.

3. Collective effects We have included the basic ionic screening effects which are somewhat different from those of other authors, and which, in contrast, reduce to well known collective corrections to photoabsorption in the limit of no magnetic field.

APPENDIX

The electron field operator $\psi(\mathbf{r},t)$ is built in cylindrical coordinates from the states of the theory, in the usual way,

$$\psi(\mathbf{r},t) = \textstyle\sum_{n,s}\sum_p L^{-1/2} a_{n,s}(p) e^{ipz} \times e^{-i(p^2/2m+n\omega_c)t} u_{n,s}(|\mathbf{r}_\perp|) e^{-i(n-s)\phi} \quad (54)$$

where p is the momentum in the direction of the field $a_{n,s}(p)$ is the annihilation operator for the indicated mode, and $\epsilon_p = p^2/2m$. To calculate the correlator needed in (23),

$$\Delta_e(\mathbf{x},t) = \langle \psi^\dagger(\mathbf{r},t)\psi(\mathbf{r},t)\psi^\dagger(\mathbf{0},0)\psi(\mathbf{0},0)\rangle\,, \quad (55)$$

we first write

$$\psi(\mathbf{r},t)\psi^\dagger(\mathbf{0},0) = -\psi^\dagger(\mathbf{0},0)\psi(\mathbf{r},t) + C(\mathbf{r},t) \quad (56)$$

where the anticommutator function C is a c-number. In the present work we are considering only non-degenerate electrons and therefore we discard the first term on the RHS of (56), since its contribution is of higher order in the fugacity of the electrons, $e^{\beta\mu_e}$, i.e., a correction for Fermi statistics. (Note that the function C itself would be unchanged for the case of Bose statistics, while the discarded term would be of the other sign.)

Before proceeding further we note that since the functions $u_{n,s}(\mathbf{r}_\perp)$ for $n \neq s$ vanish at $\mathbf{r}_\perp$), and one electron field is evaluated at $\mathbf{r}_\perp$) in each expression that we encounter, only terms with $n = s$ will enter, and the azimuthal angle ϕ will not appear in any expressions. Henceforth we use radial functions labeled with the index n alone; $u_n(\rho) \equiv u_{n,n}(\rho)$.

Explicitly, the function C is now given by

$$C(|\mathbf{r}_\perp|,z,t) = \sum_n \int \frac{dp}{2\pi} e^{ipz} e^{-i[p^2/(2m)+n\omega_c]t} u_n(|\mathbf{r}_\perp|) u_n(0) \quad (57)$$

For the remainder of the evaluation of (55) and its Fourier transform we need the thermal expection value,

$$\langle [a_n(p)]^\dagger [a_n(p')]\rangle = \delta_{n,n'}\delta_{p,p'} e^{\beta\mu_e} e^{-(p^2/2m+n\omega_c)\beta}\,, \quad (58)$$

where to determine the fugacity $\exp[\beta\mu_e]$ to be used in (58) we calculate the electron density (for convenience at $r = 0$) as

$$n_e = e^{\mu_e\beta} \sum_{n=0}^{\infty} |u_n(0)|^2 e^{-n\beta\omega_c} \int_{-\infty}^{\infty} \frac{dp}{2\pi} e^{-\beta p^2/(2m)} \quad (59)$$

Using $|u_n(0)|^2 = m\omega_c/(2\pi)$ we obtain,

$$e^{\beta\mu_e} = (1 - e^{-\beta\omega_{ce}}) \frac{\pi^{3/2} 2^{3/2} n_e^{(0)} \beta^{1/2}}{m^{3/2}\omega_{ce}} \quad (60)$$

tric function ϵ, where it occurs in the denominator, is taken to be that of a classical plasma. The numerators, which are not in these works identified as the imaginary part of the dielectric functions, require the quantum treatment in order to avoid an ultraviolet divergence (or the introduction on an arbitrary logarithm). In any application the full quantum form should probably be used in the denominator as well.

Putting together (56), (57), and (61) we obtain

$$\Delta_e(\mathbf{k},\omega) = e^{\beta\mu_e}\int d^2r_\perp\, dz\, e^{i\mathbf{k}_\perp\cdot\mathbf{r}_\perp}e^{i\omega t}e^{ikz} \times\sum_{j=0}^{\infty}\int\frac{dp}{2\pi}e^{ipz}u_j(|\mathbf{r}_\perp|)u_j(0)C(\mathbf{r}_\perp,z,t) \qquad (61)$$

For examination of behavior in the region of the main cyclotron resonance it suffices to include only the terms $j = 0, 1$ in the sum in (61) and terms with $n = 0, 1$ in the sum in(57). Inserting

$$u_0(\rho) = \sqrt{\frac{\omega_c m}{2\pi}}e^{-m\omega_c\rho^2/4}\ ,$$
$$u_1(\rho) = \frac{m\omega_c}{2\sqrt{\pi}}\rho e^{-m\omega_c\rho^2/4} \qquad (62)$$

where $\gamma = \omega_c m/2$, and performing the Fourier transforms yields the result (26).

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